Exercise 24

Regard y as the independent variable and x as the dependent variable and use implicit differentiation to find dx/dy.

$$y \sec x = x \tan y$$

Solution

Differentiate both sides of the given equation with respect to y.

$$\frac{d}{dy}(y \sec x) = \frac{d}{dy}(x \tan y)$$

$$\left[\frac{d}{dy}(y)\right] \sec x + y \left[\frac{d}{dy}(\sec x)\right] = \left[\frac{d}{dy}(x)\right] \tan y + x \left[\frac{d}{dy}(\tan y)\right]$$

$$(1) \sec x + y \left[(\sec x \tan x) \cdot \frac{d}{dy}(x)\right] = (x') \tan y + x(\sec^2 y)$$

$$\sec x + x' y \sec x \tan x = x' \tan y + x \sec^2 y$$

Solve for x'.

$$x'(y \sec x \tan x - \tan y) = x \sec^2 y - \sec x$$

$$x' = \frac{x \sec^2 y - \sec x}{y \sec x \tan x - \tan y}$$